4.4a Modeling and Optimization

Strategy for solving max/min problems

1. Understand the problem. — which one is being optimized
2. Use pictures, label variables, constants. Find a function to model the problem.
3. Graph the function. Find the domain that makes sense
4. Find the critical points and endpoints (candidate)
5. Use the first or second derivative test to identify maximums and minimums.
6. Answer the original question.

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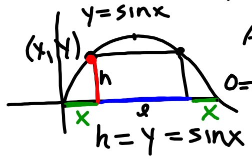
A rectangle is to be inscribed under one arch of a sine curve. What is the largest area the rectangle can have, and what dimensions give that area?

1. guess and check with 4.4 rectangle under sine curve.tns



$A = \text{Area} = l \cdot h$

$h = y = \sin x$

$x + l + x = \pi$

$l = \pi - 2x$

$\pi - 2x = 0$

$x = \frac{\pi}{2}$

$A = (\pi - 2x)\sin x$

$0 = A' = (\pi - 2x)\cos x + \sin x(-2)$

$x = 0.710463$

$x = 0 \quad A = 0$

$x = \frac{\pi}{2} \quad A = 0$

$A(.710463) = 1.12219$

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Solve using a derivative

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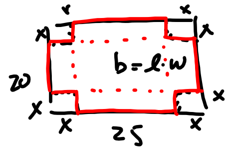
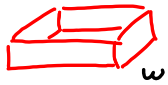
An open top box is to be made by cutting squares from the corners of a 20 by 25 inch sheet of cardboard and bending up the sides. What is the largest possible volume?

max → V = volume





$V = l \cdot w \cdot h$

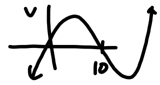
$h = x$

$w = 20 - 2x$

$l = 25 - 2x$

$V = (25 - 2x)(20 - 2x)x$

$V' = 12x^2 - 180x + 500 = 0$



$x = 3.68119 \quad V(3.68119) = 820.528$ ↑ max vol

~~$x = 11.319$~~

$x = 0 \quad V = 0$

$x = 10 \quad V = 0$

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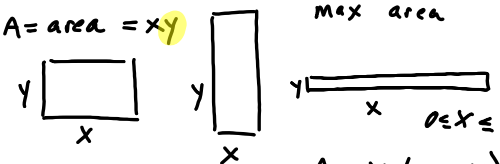
What is the largest rectangular garden that can be enclosed with 600 feet of fence?

$A = \text{area} = xy$ max area



$0 \le x \le 300$

$x = \text{width}$

$y = \text{length}$

A' + 0 − (150)

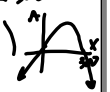
$2x + 2y = 600$

$x + y = 300$

$y = 300 - x$

$A'' = -2 < 0$

$A = x(300 - x)$



$A = 300x - x^2$

$A' = 300 - 2x = 0$

$x = 150 \quad A = 150 \cdot 150$ max

$x = 0 \quad A = 0$ max since A' change from + to −

$x = 300 \quad A = 0$

$A = 22{,}500$

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